Assignment 8

1. Consider

$$\begin{aligned} \min \quad & x_1^2 + x_2^2 \\ \text{subject to} \quad & (x_1 - 1)^2 + (x_2 - 1)^2 \leq 1, \\ & (x_1 - 1)^2 + (x_2 + 1)^2 \leq 1 \end{aligned}$$

(a) Give the feasible set and optimal solution x^*.

(b) Give the KKT conditions and explain whether there exists λ_1^*, λ_2^* such that $x^*, (\lambda_1^*, \lambda_2^*)$ satisfy the KKT conditions.

2. Consider the convex problem

$$\min f(x) \text{ subject to } g_i(x) \leq 0, i = 1, \ldots, m$$

Assume that $x^* \in \mathbb{R}^n, \lambda^* \in \mathbb{R}^m$ satisfy the KKT conditions

$$\begin{aligned} g_i(x^*) \leq 0, i = 1, \ldots, m \\ \lambda_i^* \geq 0, i = 1, \ldots, m \\ \lambda_i^* g_i(x^*) = 0, i = 1, \ldots, m \\ \nabla f(x^*) + \sum \lambda_i^* \nabla g_i(x^*) = 0 \end{aligned}$$

Show that

$$\langle \nabla f(x^*), x - x^* \rangle \geq 0$$

for all feasible x.

3. Calculate the conjugate of the following functions:

(a) Negative logarithm: $f(x) = -\sum_{i=1}^N \log x_i$;

(b) Quadratic function: $f(x) = x^T A x + b^T x + c$, where $A \in \mathbb{R}^{N \times N}$ is a symmetric positive definite matrix.

(c) Norm: $f(x) = \|x\|, x \in \mathbb{R}^N$.